| | 2 | | 4 | 5 | | 7 | | | 10 | | 12 | 13 | | 15 | | | 18 | | 20 | 21 | | 23 | | | 26 | | 28 | 29 | |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|---|
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |
| | 2 | | 4 | 5 | | 7 | | | 10 | | 12 | 13 | | 15 | | | 18 | | 20 | 21 | | 23 | | | 26 | | 28 | 29 | |
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |
| | 2 | | 4 | 5 | | 7 | | | 10 | | 12 | 13 | | 15 | | | 18 | | 20 | 21 | | 23 | | | 26 | | 28 | 29 | |
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |
| | 2 | | 4 | 5 | | 7 | | | 10 | | 12 | 13 | | 15 | | | 18 | | 20 | 21 | | 23 | | | 26 | | 28 | 29 | |
| | 2 | | 4 | 5 | | 7 | | | 10 | | 12 | 13 | | 15 | | | 18 | | 20 | 21 | | 23 | | | 26 | | 28 | 29 | |
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |
| | 2 | | 4 | 5 | | 7 | | | 10 | | 12 | 13 | | 15 | | | 18 | | 20 | 21 | | 23 | | | 26 | | 28 | 29 | |
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |
| | 2 | | 4 | 5 | | 7 | | | 10 | | 12 | 13 | | 15 | | | 18 | | 20 | 21 | | 23 | | | 26 | | 28 | 29 | |
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |
| | 2 | | 4 | 5 | | 7 | | | 10 | | 12 | 13 | | 15 | | | 18 | | 20 | 21 | | 23 | | | 26 | | 28 | 29 | |
| | 2 | | 4 | 5 | | 7 | | | 10 | | 12 | 13 | | 15 | | | 18 | | 20 | 21 | | 23 | | | 26 | | 28 | 29 | |
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |
| | 2 | | 4 | 5 | | 7 | | | 10 | | 12 | 13 | | 15 | | | 18 | | 20 | 21 | | 23 | | | 26 | | 28 | 29 | |
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |
| 1 | | 3 | | | 6 | | 8 | 9 | | 11 | | | 14 | | 16 | 17 | | 19 | | | 22 | | 24 | 25 | | 27 | | | 3 |

Harri Kuisti

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SOLVING RUBIK'S CUBE WITH EXCEL

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Harri Kuisti

Solving Rubik's Cube with Excel

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The globe is too big and far too messy, let's solve to cube first.

Modelling the Cube

There are many possibilities to model a Rubik's Cube, here is one. The squares can be numbered from 1 to 48 as shown in figure 1. How they move is presented in the Table 1.

| | | | 40 | 39 | 38 | | | | | | |
|----|----|----|----|----|----|----|-----------|----|-----------------|----|-----------|
| | | | 37 | | 36 | | | | | | |
| | | | 35 | 34 | 33 | | | | | | |
| 14 | 12 | 9 | 1 | 2 | 3 | 27 | 29 | 32 | 46 | 47 | 48 |
| 15 | | 10 | 4 | | 5 | 26 | | 31 | 44 | | 45 |
| 16 | 13 | 11 | 6 | 7 | 8 | 25 | 28 | 30 | <mark>41</mark> | 42 | 43 |
| | | | 17 | 18 | 19 | | | | | | |
| | | | 20 | | 21 | | | | | | |
| | | | 22 | 23 | 24 | | | | | | |

Figure 1. Rubik's cube as seen from above if it could be torn open and flattened as a box. The yellow bottom is here imagined as seen through the cube and with the numbers in these positions. Note that the order of numbers is different for the yellow side after turning the cube and when looking at it from outside and not through the cube.

| L | R | U | D | F | в | ľ | r' | u' | ď | f | b' | 2L | 2R | 2U | 2D | 2F | 2B |
|----|----|----|----|----|-----|----|----|----|----|----|----|----|--------------------|----------|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| | | | | | | | | - | | | | | | | | | |
| - | | | | | | | | | | | | | | | | | |
| 40 | 1 | 6 | 1 | 1 | 27 | 17 | 1 | 2 | 1 | 1 | 14 | 41 | 1 | • | 1 | 1 | 10 |
| 40 | - | 4 | - | - | 21 | 11 | - | 13 | - | + | 40 | 41 | <u></u> | 7 | - | - | 40 |
| 4 | 40 | 4 | 4 | 4 | 29 | 4 | 4 | 10 | 4 | 4 | 12 | 4 | 42 | <u> </u> | 4 | 4 | 4/ |
| 3 | 19 | 1 | 3 | 3 | 32 | 3 | 38 | 0 | 3 | 3 | 9 | 3 | 43 | 0 | 3 | 3 | 40 |
| 37 | 4 | 7 | 4 | 4 | 4 | 20 | 4 | 2 | 4 | 4 | 4 | 44 | 4 | 5 | 4 | 4 | 4 |
| 5 | 21 | 2 | 5 | 5 | 5 | 5 | 36 | 7 | 5 | 5 | 5 | 5 | 45 | 4 | 5 | 5 | 5 |
| 35 | 6 | 8 | 6 | 16 | 6 | 22 | 6 | 1 | 6 | 25 | 6 | 46 | 6 | 3 | 6 | 43 | 6 |
| 7 | 7 | 5 | 7 | 13 | 7 | 7 | 7 | 4 | 7 | 28 | 7 | 7 | 7 | 2 | 7 | 42 | 7 |
| 8 | 24 | 3 | 8 | 11 | 8 | 8 | 33 | 6 | 8 | 30 | 8 | 8 | 48 | 1 | 8 | 41 | 8 |
| 14 | 9 | 17 | 9 | 9 | 3 | 11 | 9 | 33 | 9 | 9 | 46 | 16 | 9 | 25 | 9 | 9 | 32 |
| 12 | 10 | 18 | 10 | 10 | 10 | 13 | 10 | 34 | 10 | 10 | 10 | 15 | 10 | 26 | 10 | 10 | 10 |
| 9 | 11 | 19 | 11 | 41 | 11 | 16 | 11 | 35 | 11 | 8 | 11 | 14 | 11 | 27 | 11 | 30 | 11 |
| 15 | 12 | 12 | 12 | 12 | 2 | 10 | 12 | 12 | 12 | 12 | 47 | 13 | 12 | 12 | 12 | 12 | 29 |
| 10 | 13 | 13 | 13 | 42 | 13 | 15 | 13 | 13 | 13 | 7 | 13 | 12 | 13 | 13 | 13 | 28 | 13 |
| 16 | 14 | 14 | 38 | 14 | 1 | 9 | 14 | 14 | 22 | 14 | 48 | 11 | 14 | 14 | 30 | 14 | 27 |
| 42 | 45 | 45 | 20 | 45 | 4.5 | 42 | 45 | 45 | 22 | 45 | 45 | 40 | 45 | 45 | 24 | 45 | 40 |
| 13 | 10 | 15 | 39 | 13 | 15 | 12 | 15 | 15 | 23 | 15 | 15 | 10 | 15 | 15 | 31 | 15 | 10 |
| 11 | 10 | 16 | 40 | 43 | 10 | 14 | 10 | 16 | 24 | 0 | 10 | 9 | 10 | 10 | 32 | 25 | 10 |
| 1 | 17 | 25 | 17 | 22 | 17 | 41 | 17 | 9 | 17 | 19 | 17 | 40 | 17 | 33 | 17 | 24 | 1/ |
| 18 | 18 | 26 | 18 | 20 | 18 | 18 | 18 | 10 | 18 | 21 | 18 | 18 | 18 | 34 | 18 | 23 | 18 |
| 19 | 43 | 27 | 19 | 17 | 19 | 19 | 3 | 11 | 19 | 24 | 19 | 19 | 38 | 35 | 19 | 22 | 19 |
| 4 | 20 | 20 | 20 | 23 | 20 | 44 | 20 | 20 | 20 | 18 | 20 | 37 | 20 | 20 | 20 | 21 | 20 |
| 21 | 45 | 21 | 21 | 18 | 21 | 21 | 5 | 21 | 21 | 23 | 21 | 21 | 36 | 21 | 21 | 20 | 21 |
| 6 | 22 | 22 | 14 | 24 | 22 | 46 | 22 | 22 | 30 | 17 | 22 | 35 | 22 | 22 | 38 | 19 | 22 |
| 23 | 23 | 23 | 15 | 21 | 23 | 23 | 23 | 23 | 31 | 20 | 23 | 23 | 23 | 23 | 39 | 18 | 23 |
| 24 | 48 | 24 | 16 | 19 | 24 | 24 | 8 | 24 | 32 | 22 | 24 | 24 | 33 | 24 | 40 | 17 | 24 |
| 25 | 30 | 33 | 25 | 6 | 25 | 25 | 27 | 17 | 25 | 43 | 25 | 25 | 32 | 9 | 25 | 16 | 25 |
| 26 | 28 | 34 | 26 | 26 | 26 | 26 | 29 | 18 | 26 | 26 | 26 | 26 | 31 | 10 | 26 | 26 | 26 |
| 27 | 25 | 35 | 27 | 27 | 48 | 27 | 32 | 19 | 27 | 27 | 1 | 27 | 30 | 11 | 27 | 27 | 14 |
| 20 | 24 | 20 | 20 | 7 | 20 | 20 | 26 | 20 | 20 | 42 | 20 | 20 | 20 | 20 | 20 | 12 | 20 |
| 20 | 26 | 20 | 20 | 20 | 47 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 29 | 20 | 20 | 20 | 120 |
| 23 | 20 | 29 | 29 | 29 | 4/ | 29 | 31 | 29 | 29 | 23 | 2 | 29 | 20 | 29 | 23 | 23 | 12 |
| 30 | 32 | 30 | 22 | 0 | 30 | 30 | 20 | 30 | 30 | 41 | 30 | 30 | $\frac{2}{\alpha}$ | 30 | 14 | 11 | 30 |
| 31 | 29 | 31 | 23 | 31 | 31 | 31 | 28 | 31 | 39 | 31 | 31 | 31 | 26 | 31 | 15 | 31 | 31 |
| 32 | 27 | 32 | 24 | 32 | 46 | 32 | 30 | 32 | 40 | 32 | 3 | 32 | 25 | 32 | 16 | 32 | 9 |
| 33 | 8 | 9 | 33 | 33 | 38 | 33 | 48 | 25 | 33 | 33 | 35 | 33 | 24 | 17 | 33 | 33 | 40 |
| 34 | 34 | 10 | 34 | 34 | 36 | 34 | 34 | 26 | 34 | 34 | 37 | 34 | 34 | 18 | 34 | 34 | 39 |
| 46 | 35 | 11 | 35 | 35 | 33 | 6 | 35 | 27 | 35 | 35 | 40 | 22 | 35 | 19 | 35 | 35 | 38 |
| 36 | 5 | 36 | 36 | 36 | 39 | 36 | 45 | 36 | 36 | 36 | 34 | 36 | 21 | 36 | 36 | 36 | 37 |
| 44 | 37 | 37 | 37 | 37 | 34 | 4 | 37 | 37 | 37 | 37 | 39 | 20 | 37 | 37 | 37 | 37 | 36 |
| 38 | 3 | 38 | 30 | 38 | 40 | 38 | 43 | 38 | 14 | 38 | 33 | 38 | 19 | 38 | 22 | 38 | 35 |
| 39 | 39 | 39 | 31 | 39 | 37 | 39 | 39 | 39 | 15 | 39 | 36 | 39 | 39 | 39 | 23 | 39 | 34 |
| 41 | 40 | 40 | 32 | 40 | 35 | 1 | 40 | 40 | 16 | 40 | 38 | 17 | 40 | 40 | 24 | 40 | 33 |
| 17 | 41 | 41 | 46 | 30 | 41 | 40 | 41 | 41 | 43 | 11 | 41 | 1 | 41 | 41 | 48 | 8 | 41 |
| 12 | 12 | 12 | 44 | 28 | 42 | 42 | 12 | 42 | 45 | 12 | 42 | 42 | 12 | 12 | 47 | 7 | 12 |
| +4 | 20 | 42 | 44 | 20 | 42 | 42 | 40 | 42 | 40 | 10 | 42 | 42 | 1 2 | 42 | 41 | 6 | 42 |
| 43 | 38 | 43 | 41 | 25 | 43 | 43 | 19 | 43 | 48 | 16 | 43 | 43 | 3 | 43 | 46 | 6 | 43 |
| 20 | 44 | 44 | 47 | 44 | 44 | 37 | 44 | 44 | 42 | 44 | 44 | 4 | 44 | 44 | 45 | 44 | 44 |
| 45 | 36 | 45 | 42 | 45 | 45 | 45 | 21 | 45 | 47 | 45 | 45 | 45 | 5 | 45 | 44 | 45 | 45 |
| 22 | 46 | 46 | 48 | 46 | 9 | 35 | 46 | 46 | 41 | 46 | 32 | 6 | 46 | 46 | 43 | 46 | 3 |
| 47 | 47 | 47 | 45 | 47 | 12 | 47 | 47 | 47 | 44 | 47 | 29 | 47 | 47 | 47 | 42 | 47 | 2 |
| 48 | 33 | 48 | 43 | 48 | 14 | 48 | 24 | 48 | 46 | 48 | 27 | 48 | 8 | 48 | 41 | 48 | 1 |

Table 1. How the standard moves affect the squares.

The scrambled cube can be presented as a line resembling a mountainous landscape (see figure 2.) while the solved cube is a nice slope of figure 3.



Figure 2. Scrambled cube presented as numbers of the figure 1 thrown in disarray.



Figure 3. Solved cube has the numbers in order.

The scrambled cube can be presented as a 48 x 48 -matrix with 48 ones thrown into the company of 2256 zeros so that there is one 'i' in each row and in each column (see the example of figure 4. showing a 8 x 8 -matrix for simplicitys sake).

Similar matrix can be utilized for modelling the moves. We multiply the matrix showing the positions of the cube by a matrix describing the move and get as a result a new position matrix.

| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

Figure 4. Matrix describing a scrambled Rubik's cube or a move (for complete it is of size 48 x 48, but the idea is the same).

A solved cube can be presented as identity matrix (see the figure 5.).

| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Figure 5. A solved Rubik's cube can be presented as an identity matrix of the size 48 x 48 (here just of size 8 x 8 for the sake of simplicity).

The Solution

There are roughly 43 000 000 000 000 000 000 different positions in a 3 x 3 x 3 -cube. It simply cannot be solved by trial and error -method. Something else needs to be done. There are many methods available, some for solving the real thing by turning it in your fingers (fast in 10-20 seconds or slowly and meditatively), others for solving it mathematically (in a computer or even in a mobile phone).

I realize there is no shortage in methods for solving the Rubik's cube, but I will nevertheless present my own algorithm.

I have written the code in Visual Basic as an Excelmacro. It is important to minimize the tasks given to Excel because it thinks we are interested in seeing all intermediary results as numbers in cells and consumes its energy in giving us that. Visual Basic however can be commanded to concentrate on calculations and to give us just the relevant results, which leads to fast operation.

I will not try to explain the details of the code as such. I will instead make a serious attempt at explaining how it works.

My idea is simply this: It is possible to try different series of moves without calculating all positions. If one square lands at a wrong place in the end, that series of moves does not qualify as a solution. This enables us to discard most of the sequences of moves and speeds up the process leading to one of many solutions. Yes, there are fortunately many solutions, not just one. The cube is like a labyrinth without any dead ends. It is a good metaphor for life: Hard and often a serious mess but always with many solutions.

When we hold a scrambled cube in our hands we know the outcome but have no idea which moves have brought it to that condition. The solution is to look backwards at the route of a single square at first. Here we just need to assume something and afterwards we will see if the assumption is true or not.

Fortunately a square cannot travel arbitrarily but if we choose for instance the move L (turning the orange side clockwise while looking at the green one) we know immediately where one single square has been before that move. The same holds for that position as well. Once we have chosen again to assume one standard move, we can infer the previous position, and do this the whole way back through N moves. After that we may be shocked: The series of assumptions place the square we are analysing to a completely strange location, implying an initial unsolved starting point. That will happen in most cases. We must then reject the erroneous assumptions.

We let the computer do this tedious job of testing many series of randomly chosen sequences of moves. And with good luck we have a series of moves that bring the cube to the present position starting from the solved condition of the Rubik's cube.

It is shown how the Rubik's cube can be solved with Excel. Visual Basic macros developed for this purpose are included in the book.

