TREE SUCCESSOR ALGEBRA

A new branch in mathematics

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For my family

Contents

1.Introduction

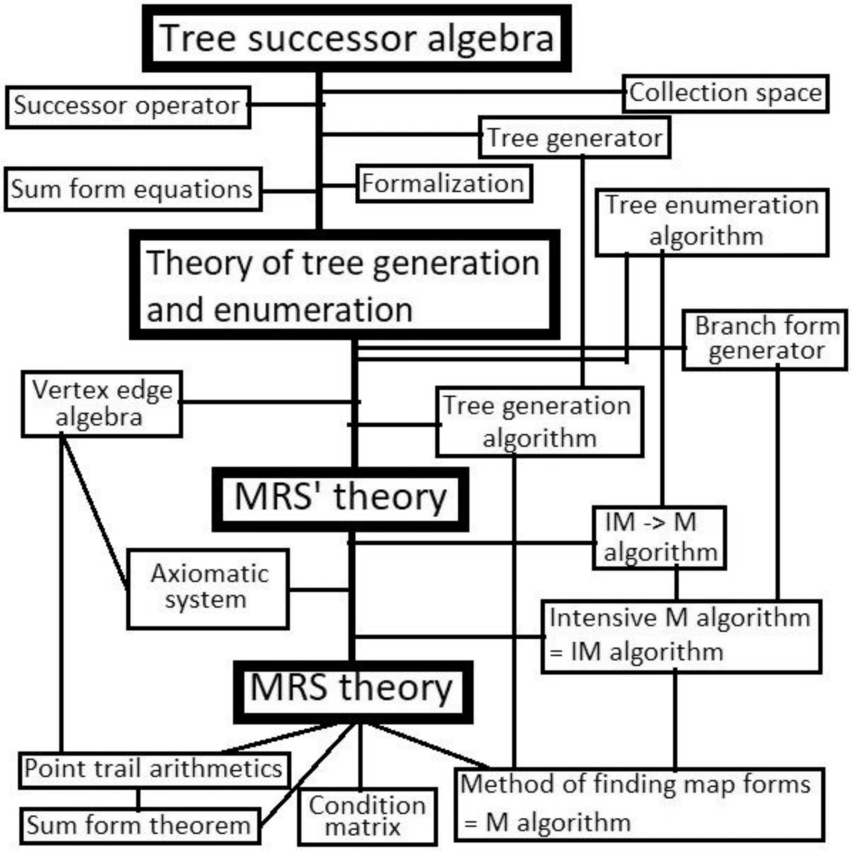
What are mathematical trees, really? One might say that they are undirected graphs with exactly one unique path between any two vertices. But what more is there? What are the hidden patterns behind their structures? Could there be a new algebraic framework that could be used to represent any tree, and even go beyond the traditional boundaries?

Tree successor algebra is a new mathematics branch which studies rooted trees – or rather – sum forms; algebraic representations of the trees' graphical structures, such that the graphical representation of a tree can always be deduced from the sum form and vice versa. Getting deeper in the theory shows that all the branches connected to the root vertex, can be thought of as a single object called an arbitrary collection of branches. A formal theory of tree generation being one goal for tree successor algebra, the successor operator is introduced and makes it possible to quantize tree generation processes and put them on a formal basis.

The scope of tree generation is broadened to include an axiomatic system followed by the application area of sum form equations.

Tree successor algebra has application areas in tree generation and enumeration, algebraic tree theory and computer science (file systems, databases etc.), to name a few. But this theory also has several possible connections to other branches of mathematics, for example number theory, linear algebra, algebra, and combinatorics.

Is tree generation just a process consisting of three action types, stacking of twos, forming a group of ones or both? This description becomes clearer when one becomes more familiar with this book.

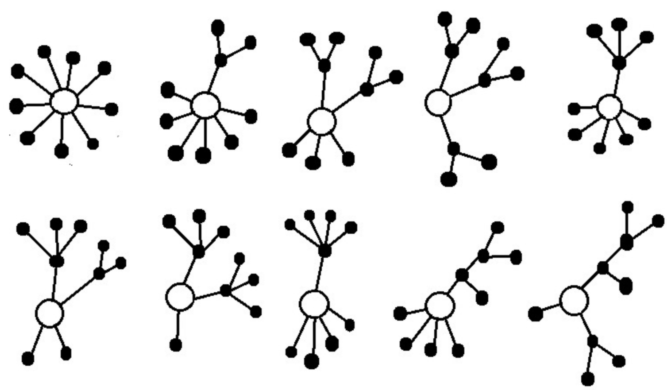

Tree successor algebra
Successor operator
Collection space
Tree generator
Sum form equations
Formalization
Tree enumeration algorithm
Theory of tree generation and enumeration
Branch form generator
Vertex edge algebra
Tree generation algorithm
MRS' theory
IM -> M algorithm
Axiomatic system
Intensive M algorithm = IM algorithm
MRS theory
Point trail arithmetics
Sum form theorem
Condition matrix
Method of finding map forms = M algorithm

2.Sum form theorem

In the year 2014 I started to research one particular problem in graph theory. The problem was from the film Good Will Hunting:

Draw all the homeomorphically irreducible trees with n = 10.

So, one had to draw all the size 10 series-reduced free trees. May sound complicated, but it actually is quite simple. The following picture has the solution:



See those objects which consist of white and black vertices and the edges which connect the vertices to each other? They are trees, series-reduced free trees in this case. What comes to this theory, the size of a tree is the number of vertices it has. As such, if you count the vertices in each tree, you will notice that every one of them has ten each and are thus size 10 trees. The white vertices may seem a bit odd among the many black ones, but they do have a purpose: They represent the so-called root. A virtual root in this case, as free trees have no real roots in mathematics.

The root / virtual root is used in my ideas as a base of a tree's structure when vertex edge algebra – the language of mathematical trees I have created – constructs the said tree. In the year 2020 I finally constructed the theorem which had already been present in my theories since 2014. Thanks to this theorem, vertex edge algebra can be used as a tree language. I call the theorem sum form theorem, as vertex edge algebra constructs trees as sums of numbers, which now represent forms of these trees.

Sum form theorem

Any nonempty, finite rooted tree can be represented as a finite string of symbols, consisting of four types at most: 1 (one), $+$ (plus sign), ((starting bracket) and) (ending bracket).

The tree generator produces all the trees, or strings of symbols, in acceptable forms. The string s of the tree represents the tree's graphical structure g, and there are two structure preserving mappings D and W.

The mapping D is the draw operator

$$D: S_r \to G_r$$

and the mapping W is the write operator

$$W: G_r \to S_r$$

, where S_r is the set of rooted tree sum forms and G_r is the set of rooted tree graphical forms.

Tree successor algebra: A new branch in mathematics is a book about a formal theory of tree generation with an axiomatic basis for a new object called collection space. The elements of this space, in other words collections, have a clear connection to rooted trees and are treated as variables in sum form equations, the application area of tree successor algebra. With connections to different branches of mathematics such as number theory, linear algebra and algebra, tree successor algebra shows a fundamental link between rooted tree generation and partition generation, establishing a well-defined order in which rooted trees are generated. This in turn makes it possible to define a successor operator, the unit of least action in tree generation, and generalize it in order to create a concept of tree sequences. Due to this, the concept of the infinite sequence of all rooted trees can be formed, and the notion of a rooted tree line, and thus the need for tools to solve sum form equations rises. The axiomatic system answers to this need.

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